Some Findings on Conceptual Development of Computational Skills

By **William L. Swart**

See graph at end of article

With the prevailing winds of the 1980s blowing "back to the basics" and with the countervailing efforts by those mathematics educators who have long been fighting rote memorization of facts and algorithms, one is a bit reluctant to report data on students' ability to compute. However, the findings reported here on achievement in addition and subtraction by first-grade children help make the case for the conceptual rather than the mechanized approach to the operations. The results may not be startling, but the instruction that brought them about does suggest some strategies for teaching computation.

It appears, to our dismay, that many people think of the basics as being little more than the ability to compute with paper and pencil to achieve 80 percent correct on tests of computational ability. This superficial view of the basics has two problems. Not only does it lead to superficial treatment, or no treatment at all, of topics that are basic to mathematical literacy, such as geometry, statistics, and probability, but it also leads to superficial treatment of the operations *themselves*. This lack of attention will affect students' later learning in such aspects of computation as—

a. underlying interpretation of the operations derived from work with physical models (0.5 percent corresponds to 1/2 of a square in a 100-square grid);
b. estimation (0.235 × 3.97 is approximately 1/4 of 4);
c. mental arithmetic (3 × 28 is three 20s and three 8s, or 60 + 24 = 84);
d. the relationships of the operations to each other (2/3 × 12 corresponds to 12 divided by 3, with that result multiplied by 2).

This article offers some conjectures, some strong hints, and even a bit of evidence from a curriculum project that we may produce a more knowledgeable student by spending a lot of time and effort on the aspects listed previously and by allowing the efficient mastery of algorithms to follow that work.

We start with some evidence. The data come from the Tricon elementary mathematics project that has been under development in the Mt. Pleasant, Michigan, schools for approximately ten years and has been accepted for publication. The project has among its goals the thorough development of the conceptual background needed to solve the exercises given earlier. The materials and methodology of the project rely heavily on the manipulation of concrete materials and on the drawing of diagrams as tools to be used by the students to do the operations with whole numbers, fractions, and decimals. (Previous articles stemming from this project, and somewhat revealing of its character, are William L. Swart's "Evaluation of Mathematics Instruction in the Elementary Classroom," *Arithmetic Teacher* 21 [January 1974]:7–13 and "Fractions vs. Decimals—the Wrong Issue," *Arithmetic Teacher* 29 [October 1981]:17–18.)

Table 1 compares the achievement of children from four first-grade classes in one school district who were taught addition and subtraction with the use of base-ten blocks, with the achievement of children from four first-grade classes in other school districts in similar socioeconomic areas who were not taught with the blocks. (Although Mt. Pleasant is a university town, the children tested here attended two elementary schools unaffected by residential concentrations of faculty.)

Something of the nature of the instruction will emerge as the discussion progresses. Refer to table 1. For exercise 1, $7 + 5 = \square$, not much difference is seen between the Tricon group's achievement and that of the control group (97 percent to 86 percent). For exercise 9,

$$\begin{array}{r} 48 \\ +26 \end{array},$$

the difference is considerable (44 percent to 5 percent). And for exercise 19,

$$\begin{array}{r} 304 \\ -129 \end{array},$$

we see a comparison of 20 percent to 0 percent. It should be noted that not much difference occurs between the two groups' achievement on exercises that are usually considered first-grade material. But considerable differences are noted for the exercises that are usually not taught until second, third,

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